Complex Analysis

Complex Derivatives

unit — IV

Complex Integral.

unit — V

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unit — IV

① Complex variable

$x+iy$ is a complex variable and it is denoted by z.

(A) $z = x+iy$
$\bar{z} = x-iy$] → Cartesian form

(B) $z = r(\cos\theta + i\sin\theta) \rightarrow$ Polar form

(C) $z = re^{i\theta}$
$\bar{z} = re^{-i\theta}$] — Exponential form

$z = x + iy$

$x \Rightarrow$ Real Part

$y \Rightarrow$ imaginary Part

$z = y + ix$

$y \Rightarrow$ Real Part

$x \Rightarrow$ imaginary Part

$z = x + iy$ or $z = y + ix$

$$\boxed{z = x + iy}$$

Function of A Complex Variable

$f(z)$ is a function of complex variable z and is denoted by w

$$w = f(z)$$

$$w = u + iv$$

or

$$f(z) = u + iv$$

$u \rightarrow (x, y)$

$v \rightarrow (x, y)$

$u \rightarrow$ Real Part

$v \rightarrow$ imaginary Part

Complex derivatives

1. $z = x + iy$

2. $\bar{z} = x - iy$

3. $z = r(\cos\theta + i\sin\theta)$

4. $\bar{z} = r(\cos\theta - i\sin\theta)$

5. $z = re^{i\theta}$

6. $\bar{z} = re^{-i\theta}$

7. $w = f(z)$

8. $w = u + iv$

9. $f(z) = u + iv.$

Continuity

The function $f(z)$ of a complex variable z is said to be continuous at the Point z_0 if for any given Positive Number ϵ we can find a Number δ or Δ Such that $|f(z) - f(z_0)| < \epsilon$

for all Points z of the domain Satisfying

$$|z - z_0| < \delta$$

$f(z)$ is said to be Continuous at $z = z_0$ if

$$\lim_{z \to z_0} f(z) = f(z_0)$$

Differentiability

Let $f(z)$ be a single valued function of the variable z, then

$$f'(z) = \lim_{\Delta z \to 0} \frac{f(z+\Delta z) - f(z)}{\Delta z}$$

or

$$f'(z) = \lim_{z \to z_0} \frac{f(z+z_0) - f(z_0)}{(z+z_0)}$$

Provided that the limit exists and is independent of the path along which $\Delta z \to 0$

~~Let~~

Analytic Function

A function $f(z)$ is said to be analytic at a point z_0 if $f(z)$ is differentiable not only at z_0 but at every point of some neighbourhood of z_0

A function $f(z)$ is analytic in a domain if it is analytic at every point of the domain.

The point at which the function is not differentiable is called a singular point of the function.

An analytic function is also known as

~~Holom~~ (1) Holomorphic
(2) Regular
(3) Monogenic.

Differentiable = Analytic function

Analytic function = Differentiable

= Continuous (function)

$f(z)$

Differentiable

⇓

$$f'(z) = \lim_{z \to z_0} \frac{f(z+z_0) - f(z_0)}{(z+z_0)}$$

Differentiable at the point $z = z_0$

Analytic function all point

Theorem (Analytic function)

① Necessary ② Sufficient condition

The Necessary condition for $f(z)$ to be Analytic function

Theorem :- The necessary condition for a function $f(z) = u + iv$ to be analytic at all the point in a region R are

$$f(z) = u + iv$$

$u \rightarrow (x, y)$, $v \rightarrow (x, y)$

$\frac{\partial u}{\partial x}$, $\frac{\partial u}{\partial y}$

$\frac{\partial v}{\partial x}$, $\frac{\partial v}{\partial y}$

find derivatives

① $\frac{\partial u}{\partial x}$, ② $\frac{\partial u}{\partial y}$

③ $\frac{\partial v}{\partial x}$, ④ $\frac{\partial v}{\partial y}$.

exist

(1) $\boxed{\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}}$

(2) $\boxed{\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}}$

Cauchy Riemann equations or C-R equations

Then the C-R equation satisfied

Now $f(z)$ is Analytic function

* Sufficient condition For f(z) to be Analytic

Theorem :- The sufficient condition for a function $f(z) = u + iv$ to be analytic at all the Points in a Region R are

(1) $f'(z) = \frac{\partial u}{\partial x} + i \frac{\partial v}{\partial x}$

(2) $f'(z) = \frac{\partial v}{\partial y} - i \frac{\partial u}{\partial y}$

Necessary Condition

⇓

C-R. equation

$$\boxed{\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}} \quad \boxed{\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}}$$

only check

f(z) is analytic function Yes or No

C-R equation are satisfied [Yes f(z) analytic function]

C-R equation are Not satisfied [f(z) is No analytic function]

Sufficient condition

⇓

only use find $f'(z)$

if any one

① $f'(z) = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}$

or

② $f'(z) = \frac{\partial u}{\partial y} - i\frac{\partial v}{\partial y}$