(2)

Cauchy's Integral Theorem

if a function $f(z)$ is analytic and its derivative $f'(z)$ continuous at all points inside and simple closed curve C, then

$$\int_C f(z)\, dz = 0.$$

Note :- if there is no pole inside and on the contour then the value of the integral of the function is zero.

1. "The pole at $z = z_0$ inside the given curve (max. given circle) then given integral values is ~~[illegible]~~ any number (finite)"

2. "The pole at $z = z_0$ outside the given curve (max. given circle) Then given integral values is zero."

$\boxed{2-1}$

Question ①

Find the integral $\int_c \frac{3z^2+7z+1}{z+1}\,dz$ where c is the circle $|z| = \frac{1}{2}$

Solution

$$\int_c \frac{3z^2+7z+1}{z+1} = ? \quad \text{—— (A)}$$

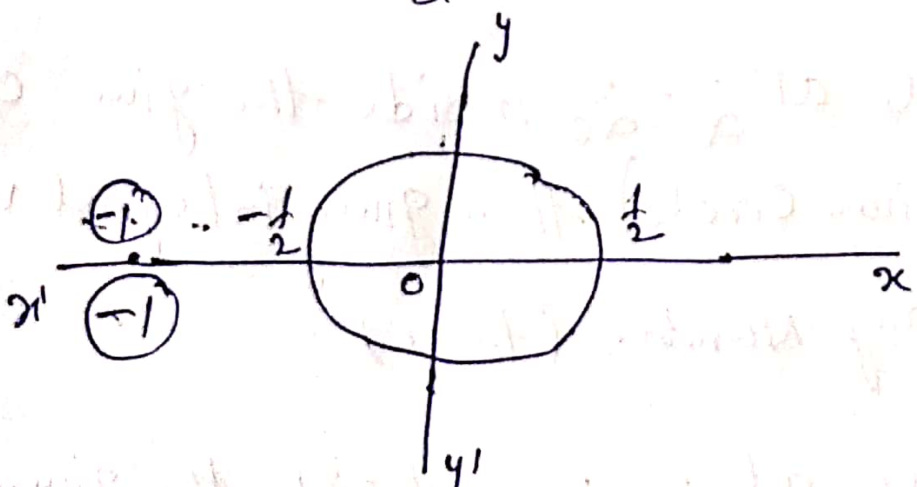
Poles of the integrand are given by Putting the Denominator equal to zero

① $z+1=0 \Rightarrow z=-1$ —— ①

find Pole is only one $\Rightarrow \boxed{\text{Poles} = -1}$

The given circle $|z| = \frac{1}{2}$ —— ②

with Centre at $z=0$ and radius $\frac{1}{2}$



Check the Poles is like inside or outside

Now at the Poles

$$z = -1$$

is like outside given circle

Then

does not enclose any singularity of the given function

$$\int_c \frac{3z^2+7z+1}{z+1}\, dz = 0.$$

Question (2)

③

Extension of Cauchy's Theorem to multiple Connected Region

if $f(z)$ is analytic in the region R between two simple closed curve C_1 and C_2 then

$$\int_{C_1} f(z)\,dz = \int_{C_2} f(z)\,dz$$

(4)

Cauchy Integral formula.

if $f(z)$ is analytic with in and on a closed curve C, and if a is any Point within C, then

$$\left[f(z_0)\right]_{z_0} = \frac{1}{2\pi i}\int_{z=z_0} \frac{f(z)}{z-z_0}\,dz.$$

Now $z = z_0$

Then z_0 is Pole.

Pole is any Number

Pole :- Find the Poles of integrand are given by Putting the denominator equal to zero

Ex $\int_c \frac{N}{D}\,dx \Rightarrow \int \frac{f(z)}{z-z_0}$ $\Rightarrow$ using denominator $z-z_0=0$

$z=z_0 \rightarrow$ Pole

Question (3)

Evaluate $\int_c \left(\frac{z}{z-2}\right) dz$ where c is the circle $|z-2| = \frac{1}{2}$

Solution :-

we have

$$\int_c \frac{z}{z-2}\, dz$$

The Poles are determined by Putting the denominator equal to zero

$$z-2=0$$
$$z=2$$

$$\boxed{\begin{aligned} z-z_0&=0\\ z&=z_0\\ z&=2\\ z_0&=2 \end{aligned}}$$

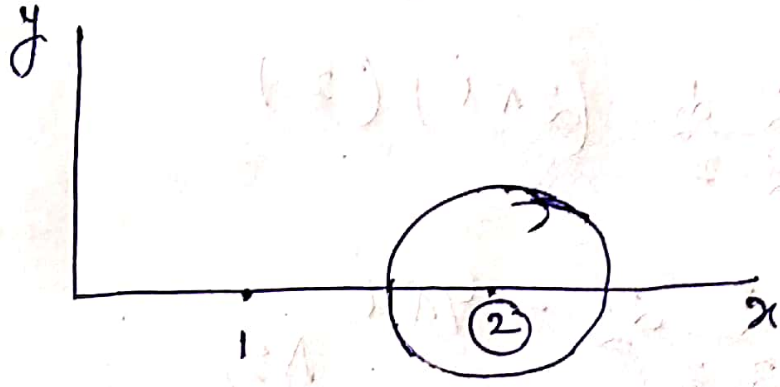
How many Pole = only one Pole is given

$z=2$

So there ~~are~~ is one Pole $z=2$

~~with centre at~~

$|z-2| = \frac{1}{2} \Rightarrow$



There is only one Pole at $z=2$ inside the given circle

(Now find values)

Now using Cauchy Integral formula

$$\left[f(z)\right]_{z_0} = \frac{1}{2\pi i}\int \frac{f(z)}{z-z_0}\,dz$$

$$\int \frac{f(z)}{z-z_0}\,dz = \left[f(z)\right]_{z_0}(2\pi i) \quad \text{—— (A)}$$

x

$$\int \frac{z}{z-2}\,dz = ?$$

$$\int \frac{f(z)}{z-z_0}\,dz = (2\pi i)\left[f(z)\right]_{z_0}$$

$$\int \frac{z}{z-2}\,dz = (2\pi i)\left[z\right]_2$$

$$\int \frac{z}{z-2}\,dz = (2\pi i)(2)$$

$$\int \frac{z}{z-2}\,dz = 4\pi i$$

As//