

Unit - V Complex Integral

1. Line Integral.

Introduction

$$z = x + iy \quad \text{--- (1)}$$

\Downarrow

$$dz = dx + i dy \quad \text{--- (2)}$$

$$dz = dx \quad \text{if } y=0 \quad \text{--- (3)}$$

$$dz = i dy \quad \text{if } x=0 \quad \text{--- (4)}$$

Now

$$dz = dx + i dy$$

$x=y$ maybe then $dx = dy$

$$dz = dx + i dx$$

$$dz = (1+i) dx$$

$$dz = dy + i dy$$

$$dz = (1+i) dy$$

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1. Line Integral

** formula. **

$$\text{Line Integral} = \int_c f(z) dz.$$

$$f(z) = u + i v.$$

$$dz = dx + i dy$$

$$= \int_c [u + i v] [dx + i dy]$$

$$= \int_c [u dx + i v dx + i u dy - v dy]$$

$$= \int_c \left[\int (u dx - v dy) + i \int (v dx + u dy) \right]$$

$$= \int_c (u dx - v dy) + i \int_c (v dx + u dy)$$

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Question No - 1

Evaluate $\int_0^{1+i} (x^2 - iy) dz$ along the path $y=x$

Solution: Along the line $y=x$

$$dy = dx$$

$$\text{Now } dz = dx + i dy$$

$$dz = dx + i dx$$

$$dz = (1+i) dx$$

then

$$= \int_0^{1+i} (x^2 + iy) dz$$

on putting $y=x$ and $dz = (1+i) dx$

$$= \int_0^1 (x^2 - ix) (1+i) dx$$

$$= \int_0^1 (1+i) (x^2 - ix) dx$$

$$= (1+i) \int_0^1 (x^2 - ix) dx$$

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$$= (1+i) \left[\frac{x^3}{3} - \frac{i}{2} x^2 \right]_0^1$$

$$= (1+i) \left[\left\{ \frac{1}{3} - \frac{i}{2} \right\} - \left\{ 0 \right\} \right]$$

$$= (1+i) \left[\frac{1}{3} - \frac{i}{2} \right]$$

$$= \frac{(1+i)(2-3i)}{6}$$

$$= \frac{2+2i-3i-3i^2}{6}$$

$$= \frac{2-i+3}{6}$$

$$= \frac{5-i}{6}$$

$$= \frac{5}{6} - \frac{i}{6}$$

Ans

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Question No - 2

Evaluate $\int_0^{1+i} (x^2 - iy) dz$ along the Parth $y = x^2$

Solution: Along the Parabola

$$y = x^2$$

$$dy = 2x dx$$

Now $dz = dx + i dy$

$$dz = dx + 2ix dx$$

$$dz = (1 + 2ix) dx$$

then

$$= \int_0^{1+i} (x^2 - iy) dz$$

$$= \int_0^1 (x^2 - ix^2) (1 + 2ix) dx$$

$$= \int_0^1 (1-i)x^2 (1 + 2ix) dx$$

$$= (1-i) \int_0^1 x^2 (1 + 2ix) dx$$

$$= (1-i) \int_0^1 (x^2 + 2ix^3) dx$$

$$= (1-i) \left[\frac{x^3}{3} + 2i \frac{x^4}{4} \right]_0^1$$

$$= (1-i) \left[\frac{x^3}{3} + i \frac{x^4}{2} \right]_0^1$$

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$$= (1-i) \left[\frac{1}{3} + \frac{i}{2} \right]$$

$$= \frac{(1-i)(2+3i)}{6}$$

$$= \frac{2-2i+3i-3i^2}{6}$$

$$= \frac{2-2i+3i+3}{6}$$

$$= \frac{5+i}{6}$$

$$= \frac{5}{6} + \frac{i}{6}$$

Ans //

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