4 ****

Milne Thomson Method

Working Rule: To Construct an analytic function by Milne Thomson Method

Case – I –: When u is given

Step ①: Find $\frac{\partial u}{\partial x}$ and rewrite it to $f_x(x,y)$

② : Find $\frac{\partial u}{\partial y}$ and rewrite it to $f_y(x,y)$

③ : Replace x by z and y by 0 in $f_x(x,y)$ to get $f_x(z,0)$

$$\boxed{f_x(x,y) = f_x(z,0)}$$

④ : Replace x by z and y by 0 in $f_y(x,y)$ to get $f_y(z,0)$

$$f_y(x,y) = f_y(z,0)$$

⑤ : find $f(z)$ by formula

$$f(z) = \int \left[f_x(z,0) - i f_y(z,0) \right] dz.$$

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Case – II

When v is given

Step-I :- Find $\frac{\partial v}{\partial x}$ and equate it to $f_x(x,y)$

(2) :- Find $\frac{\partial v}{\partial y}$ and equate it to $f_y(x,y)$

(3) :- Replace x by z and y by 0 in $f_x(x,y)$ to get $f_x(z,0)$

(4) :- Replace x by z and y by 0 in $f_y(x,y)$ to get $f_y(z,0)$

(5) : Find $f(z)$ by the formula

$$f(z) = \int [f_y(z,0) + i f_x(z,0)]\, dz.$$

4 – (2)

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Example ①

if $u = x^2 - y^2$ find a corresponding analytic function.

Solution: $u = x^2 - y^2$ — ①

$$\frac{\partial u}{\partial x} = 2x$$

$$\frac{\partial u}{\partial x} = f_x(x, y)$$

$f_x(x, y) = 2x$ — ②

Replace $x = 3$ & $y = 0$

$f_x(3, 0) = 2\cancel{3}2$ — Ⓐ

$$\frac{\partial u}{\partial y} = -2y$$ — ③

$$\frac{\partial u}{\partial y} = f_y(x, y)$$

$f_y(x, y) = -2y$ — ④

Replace $x = 3$ & $y = 0$

$f_y(x, y) = 0$ — Ⓑ

4-3

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Find $f(z)$ by the formula

$$f(z) = \int [f_x(z,0) - i f_y(z,0)]\, dz$$

$$= \int [2z - i(0)]\, dz$$

$$= \int 2z\, dz$$

$$= 2\int z\, dz$$

$$= \cancel{2}\left[\frac{z^2}{\cancel{2}}\right]$$

$$f(z) = z^2.$$

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Example (2)

if $u = e^x[x\cos y - y\sin y]$ and $f(z) = u + iv$ is an analytic function of $z = x + iy$ find $f(z)$ in terms of z by Milne Thomson Method.

Solution.

$$u = e^x(x\cos y - y\sin y) \quad \text{--- (1)}$$

$$u = e^x x\cos y - e^x y\sin y$$

$$\frac{\partial u}{\partial x} = e^x x\cos y + e^x\cos y - e^x y\sin y \quad \text{--- (2)}$$

$$\frac{\partial u}{\partial x} = f_x(x,y)$$

$$f_x(x,y) = e^x . x\cos y + e^x\cos y - e^x y\sin y$$

Replace $x = z$ & $y = 0$

$$f_x(z,0) = e^z . z\cos 0 + e^z . \cos 0 - e^z(0).\sin 0$$

$$f_x(z,0) = ze^z + e^z \quad \text{--- (3)}$$

$$\frac{\partial u}{\partial y} = xe^x(-\sin y) - e^x[y\cos y + \sin y] \quad \text{--- (4)}$$

$$\frac{\partial u}{\partial y} = f_y(x,y)$$

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$f_y(x,y) = -xe^x \sin y - e^x[y\cos y + \sin y]$

Replace $x=3$ & $y=0$

$f_y(3,0) = -3e^3 \sin 0 - e^3[(0)\cos 0 + \sin 0]$

$f_y(3,0) = 0$.

Find $f(z)$ by the formula.

$f(z) = \int [f_y(z,0) + i f_x(z,0)]\,dz$

$f(z) = \int [0 + i\{ze^z + e^z\}]\,dz$

$f(z) = \int [e^z z + e^z]\,dz$

$f(z) = \int e^z z\,dz + \int e^z dz$

$= \left[z\int e^z - \int\left\{\frac{d}{dz}z \int e^z dz\right\}dz\right] + \int e^z dz$

$= ze^z - \int e^z dz + e^z$

$= ze^z - \cancel{e^z} + \cancel{e^z}$

$f(z) = ze^z$

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