Question

if $u-v = (x-y)\left[x^2+4xy+y^2\right]$ and $f(z) = u+iv$ is an analytic function of $z = x+iy$ find $f(z)$ in terms of z.

Solution:. we know that

$$f(z) = u+iv \quad \text{—— (1)}$$

both side (i) mul^n.

$$if(z) = i\left[u+iv\right]$$

$$if(z) = ui + i^2 v$$

$$if(z) = ui - v \quad \text{—— (2)}$$

eq^n (1) + eq^n (2)

$$f(z) + if(z) = u+iv + ui - v$$

$$\left[1+i\right] f(z) = (u-v) + i(u+v)$$

Now

P—(1)

$F(z) = (1+i) f(z)$

$U = u - v$

$V = u + v$

Then

$$F(z) = U + iV \text{ ——— (A)}$$

$U \to$ Real

Then U is given

$V \to I$

$$U = (x-y)\left[x^2+4xy+y^2\right] \text{ ——— (B)}$$

$$\frac{\partial U}{\partial x} = (x-y)\left[2x+4y\right] + \left[x^2+4xy+y^2\right] (1)$$

$$\frac{\partial U}{\partial x} = (x-y)\left[2x+4y\right] + (x^2+4xy+y^2]$$

$$\frac{\partial U}{\partial x} = f_x(x,y)$$

$$f_x(x,y) = (x-y)(2x+4y) + (x^2+4xy+y^2)$$

Replace $x = 3$ & $y = 0$

$$f_x(3,0) = (3-0)\left[2\cdot 3+4(0)\right] + \left(3^2+4\cdot 3(0)+(0)^2\right]$$

P — (2)

$f_x(z,0) = z(2z) + z^2)$

$= 2z^2 + z^2$

$= 3z^2$

again

$$\frac{\partial U}{\partial y} = (x-y)[4x+2y] + (x^2+4xy+y^2)(-1)$$

$$= (x-y)(4x+2y) - (x^2+4xy+y^2)$$

$$\frac{\partial U}{\partial y} = f_y(x,y)$$

$$f_y(x,y) = (x-y)(4x+2y) - (x^2+4xy+y^2)$$

Replace $x=z$, & $y=0$

$$f_y(z,0) = (z-0)[4z+2(0)] - [z^2+4z(0)+(0)^2]$$

$$= z(4z) - z^2$$

$$= 4z^2 - z^2$$

$$= 3z^2$$

Now using M-T Method formula if U is given.

$$F(z) = \int [f_x(z,0) - i f_y(z,0)]\, dz$$

$$F(z) = \int [3z^2 - i\,3z^2]\, dz$$

$$F(z) = 3(1-i)\int z^2\, dz$$

$$F(z) = \cancel{3}(1-i)\left[\frac{z^3}{\cancel{3}}\right]$$

$$F(z) = (1-i)\, z^3$$

$$F(z) = (1+i)\, f(z)$$

$$(1+i)\, f(z) = (1-i)\, z^3$$

$$f(z) = \frac{(1-i)}{(1+i)}\, z^3 \quad \text{———} \quad (c)$$

$$f(z) = \frac{(1-i)\,(1-i)}{(1+i)\,(1-i)}\, z^3$$

$$= \frac{(1-i)^2}{1-i^2}\, z^3$$

$$= \frac{(1-i)^2}{1+1}\, z^3$$

$$f(z) = \frac{(1+i^2-2i)\,z^3}{2}$$

$$f(z) = \frac{(\cancel{1}-\cancel{1}-2i)\,z^3}{2}$$

$$f(z) = \frac{-\cancel{2}i}{\cancel{2}}\,z^3$$

$$f(z) = -iz^3$$

Ans//