5 ****

Transformation or Mapping

Transformation or Mapping :- For every Point (x, y) in the z-Plane the relation $w = f(z)$ defines a corresponding Point (u, v) in the w - Plane. we call this transformation or mapping of z-Plane into w-Plane if a Point z_0 maps in to the Point w_0, w_0 is also known as the image of z_0

if the Point $P(x, y)$ moves along a curve C in z-Plane the Point $P'(u, v)$ will move along a corresponding curve C' in w-Plane, then we say that a curve C in the z-Plane is mapped into the corres-ponding curve C' in the w-Plane by the relation.

$$\boxed{w = f(z)}$$

Example-I

Transform the rectangular region ABCD in z-Plane bounded by $x=1$, $x=3$, $y=0$ and $y=3$. Under the transformation $w=z+(2+i)$

Solution :

$$w = z + (2+i) \quad \text{—— (1)}$$

we know

$$w = u+iv$$

$$z = x+iy$$

$$w = z + (2+i) \quad \text{—— (1)}$$

$$u+iv = (x+iy) + (2+i)$$

$$u+iv = x + iy + 2+i$$

$$u+iv = (x+2) + i(y+1)$$

$$u = (x+2)$$

$$v = (y+1)$$

Real Part

z-Plane	w-Plane
x	$u = x + 2$
$x = 1$	$u = 1 + 2 = 3$
$x = 3$	$u = 3 + 2 = 5$

Imaginary Part

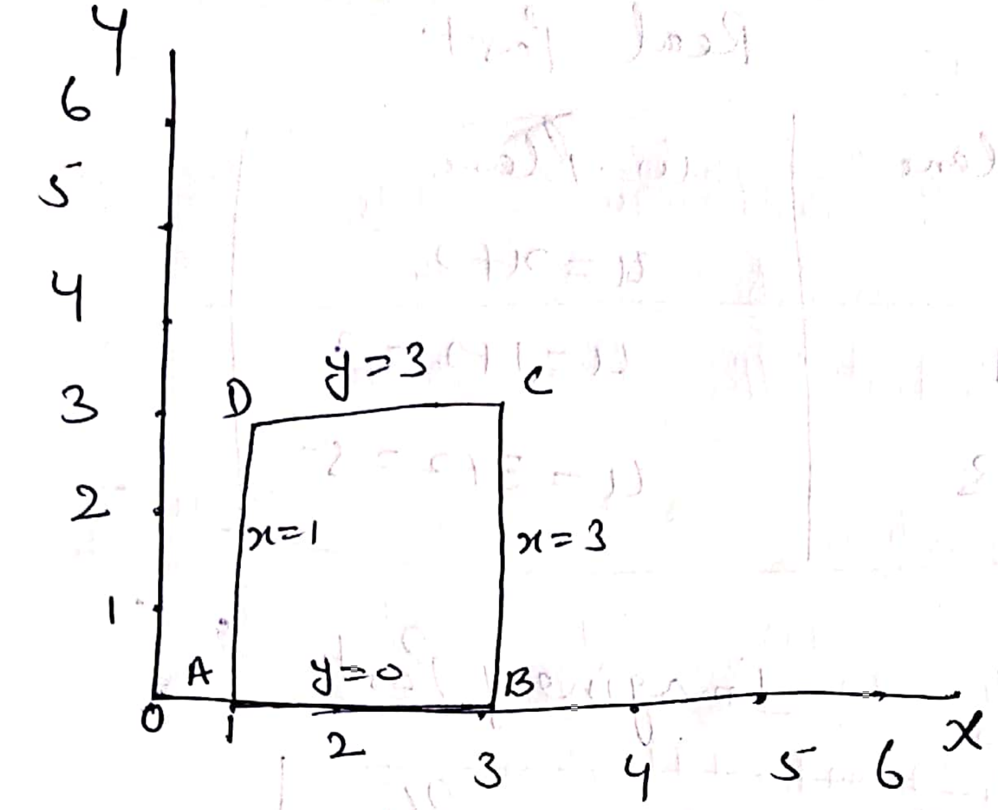
z-Plane	w-Plane
y	$v = y + 1$
$y = 0$	$v = 0 + 1 = 1$
$y = 3$	$v = 3 + 1 = 4$

$x = 1$ $\quad u = 3$

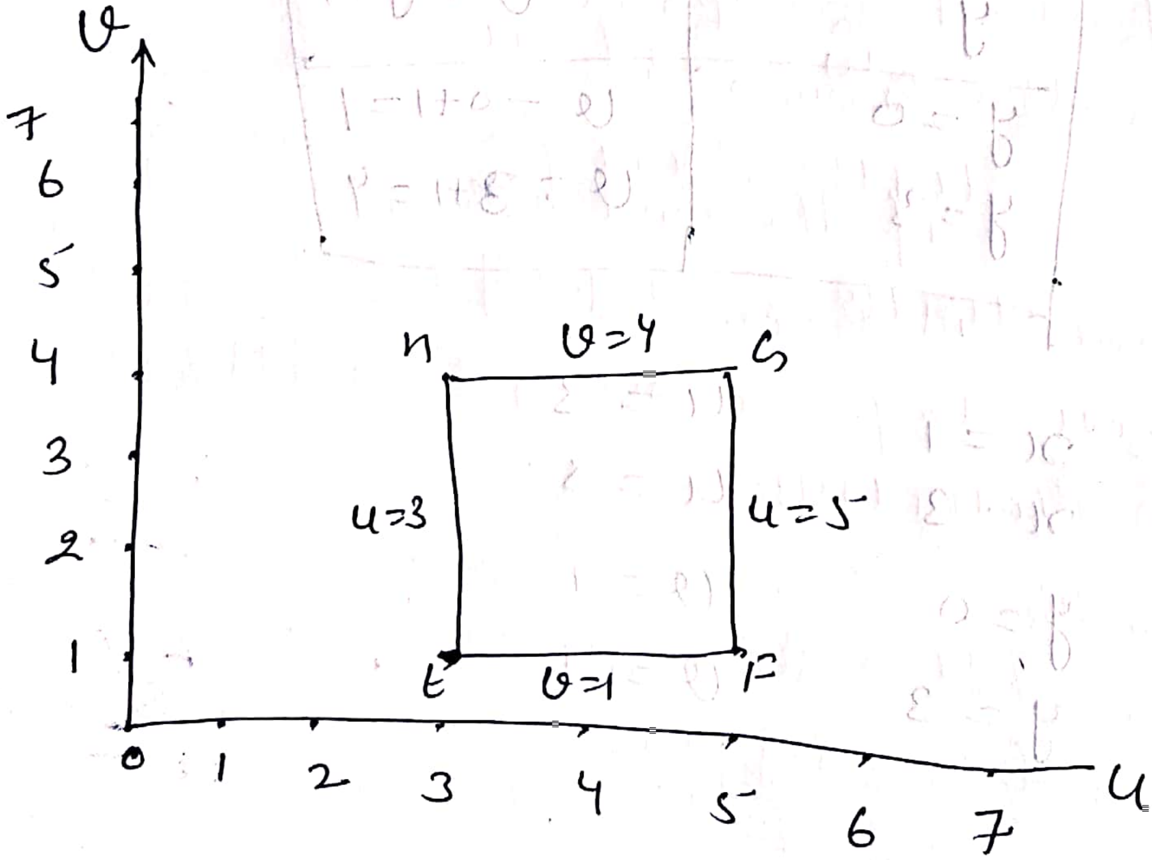
$x = 3$ $\quad u = 5$

$y = 0$ $\quad v = 1$

$y = 3$ $\quad v = 4$



z - Plane



w - Plane

(1) Translation

$$\boxed{w = z + c}$$

where $c = a + ib$

(2) Rotation

$$\boxed{w = z e^{i\theta}}$$

(3) Magnification

$$\boxed{w = cz}$$

(4) Magnification and Rotation

$$\boxed{w = cz}$$

(5) Inversion and Reflection

$$\boxed{w = \frac{1}{z}}$$

(6)