(2)

Show that the function $e^x(\cos y + i \sin y)$ is an analytic function find its derivative

$$f(z) = e^x(\cos y + i \sin y)$$

we know that

$$w = f(z)$$
$$w = u + iv$$
$$f(z) = u + iv$$

Now

$$u + iv = e^x(\cos y + i \sin y)$$

$$u + iv = e^x \cos y + i e^x \sin y$$

$$u = e^x \cos y \quad \& \quad v = e^x \sin y$$

| | |
|---|---|
| (i) $u = e^x \cos y$ | (ii) $v = e^x \sin y$ |
| $\frac{\partial u}{\partial x} = e^x \cos y$ | $\frac{\partial v}{\partial x} = e^x \sin y$ |
| $\frac{\partial u}{\partial y} = -e^x \sin y$ | $\frac{\partial v}{\partial y} = e^x \cos y$ |

use the C-R equation.



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C-R equation

$$\boxed{\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}} \quad \& \quad \boxed{\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}}$$

① $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$

$e^x \cos y = e^x \cos y$

② $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$

$-e^x \sin y = -e^x \sin y$

These are C-R equation are satisfied
Now $f(z)$ is analytic function
Hence $e^x[\cos y + i \sin y]$ is analytic

find $f'(z) = ?$

$$f'(z) = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}$$
$$= e^x \cos y + i e^x \sin y$$
$$= e^x[\cos y + i \sin y]$$
$$= e^x \cdot e^{iy}$$
$$= e^{x+iy} \quad = e^z \Rightarrow f(z) = e^z$$

Q-2-2