

Find the bilinear Transformation which map the points $z=0, -1, i$ on to $w=1, 0, \infty$
Also find the image of the unit circle

$$|z|=1$$

Find

$$w = \frac{z+1}{z-i} \quad \text{--- A}$$

we know

$$w = \frac{az+b}{cz+d} \quad \text{--- B}$$

Then $a=1, b=1, c=1, d=-i$

we know

$$z = \frac{-dw+b}{cw-a}$$

$$z = \frac{i'w+1}{w-1}$$

$$|z|=1$$

$$\left| \frac{i'w+1}{w-1} \right| = 1$$

$$|1+i\omega| = |\omega-1|$$

$$|1+i(u+i\omega)| = |u+i\omega-1|$$

$$|1+ui+i^2\omega| = |u+i\omega-1|$$

$$|1+ui-\omega| = |u+i\omega-1|$$

$$|(1-\omega)+iu| = |(u-1)+i\omega|$$

$$(1-\omega)^2 + u^2 = (u-1)^2 + \omega^2$$

$$1 + \omega^2 - 2\omega + u^2 = u^2 + 1 - 2u + \omega^2$$

$$-2\omega = -2u$$

$$\omega = u$$

$$\boxed{u = \omega}$$